A New Stochastic Multi-criteria Decision Analysis Tool based on ELECTRE III Method

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ABSTRACT

Making decisions under conditions of uncertainty is a challenging task, yet is not uncommon when engineering or management alternatives are evaluated through criteria of social and environmental sustainability. ELECTRE III is a widely-used multi-criteria decision method for ranking alternatives. The method accounts for uncertainty in criteria values by including pseudo-criteria; usually, an indifference, preference and veto thresholds are set for each criteria. Decision makers are also required to provide criteria weights signify each criterion's importance. However, choosing realistic values for each threshold remains a difficult task. Moreover, criteria weightings are known to be a major factor affecting the end result of the ranking exercise. This paper, presents a modified version of ELECTRE III, called ELECTRE-SS, that utilises stochastic techniques to account for uncertainty in the weightings and threshold values of criteria. The method can be particularly useful when a large number of stakeholders are involved in the decision-making process. The method is described and validated and its scope illustrated thorough case problems.

Background

ELECTRE III method is a well known multi-criteria decision aid (MCDA) tool. It was first introduced by Roy and co-workers (Roy 1978; Roy and Bouyssou 1986). The method was subsequently used to solve real-life problems (Duckstein, Treichel et al. 1994; Netto, Parent et al. 1996; Georgopoulou, Lalas et al. 1997; Hokkanen and Salminen 1997; Rogers 2000).

ELECTRE III uses a pseudo-criterion with indifference and preference thresholds to account for imprecision and uncertainty in the data (Rogers et al, 2000). Criteria evaluations for each alternative, criteria weightings and threshold evaluations must be specified (Karagiannidis and Moussiopoulos 1997) in order to rank alternatives.

The indifference (q) and preference (p) thresholds of any criterion can be interpreted as the minimum imprecision and the maximum margin of error respectively (Rogers et al., 2000).

ELECTRE III comprises two major phases:

1) Construction of outranking relation
2) Exploitation of the outranking relation.

In the application of ELECTRE III, both p and q values are treated as fixed values. However, this involves not only the estimation of the error in each criterion but also subjective input of the decision maker (Rogers and Bruen 1998, a). Rogers and Bruen (1998) described a methodology for selecting realistic threshold values for use in environmental appraisals. The method takes into account the effect of the differences between criterion scores on human beings. The use of stochastic dominance method to determine threshold values of ELECTRE III was investigated as well (Nowak, 2004).

Most multi-criteria methods are developed with a single stakeholder in mind (Proctor and Qureshi, 2005). Therefore, it is difficult to accommodate situations where criteria weights are uncertain. ELECTRE III assumes that criteria weights are deterministic values. Lahdelma et al. (2000) noted that current techniques to elicit weights for criteria can be used with several consistency checks when the number of decision makers is small. However, the search for the right weights is particularly difficult when the number of decision makers is large. Therefore, applying ELECTRE III in situations where decision makers have incomplete preferences or in cases of multi-stakeholders with conflicting preferences is difficult without resorting to an external method to transform the weights into deterministic values (Figueria et al, 2005). Rogers and Bruen (1998, b) critiqued the methods available for eliciting weighting values with ELECTRE III. They highlighted the fact that due to the non-compensatory nature of ELECTRE III, using weight averages does not give a true representation of the stakeholders' preferences.

In this paper, we introduce a new stochastic method called ELECTRE-SS that modifies ELECTRE III method to allow for multi-stakeholder input through accepting criteria weights and thresholds as ranges rather than deterministic values. The method allows for the provision of probability distribution functions for each parameter. Hence, the new method addresses two concerns that may affect the validity of rankings obtained with ELECTRE III:

1- uncertainty in determining threshold values
2- imprecision in allocating weighting values by decision makers due to incomplete data or preferences.

The above makes ELECTRE-SS suitable for handling uncertainty resulting from multi-stakeholder cases.

Algorithm

ELECTRESS follows similar procedures to ELECTRE III method. ELECTRE-SS comprises two phases: outranking phase and exploitation phase. The outranking phase builds an outranking matrix by forming an outranking relation between the pairs of alternatives. The outranking matrix is then exploited in the second phase to produce a partial pre-order. The following notations are used in this paper:

C is the criteria set

$$G = \{g_1, ..., g_j, ..., g_n\}$$

A is the set of alternatives or actions

$$A = \{a_1, ..., a_i, ..., a_m\}$$

W is the criteria importance index (weight) vector

$$W = \{\overline{w}_1, ..., \overline{w}_j, ..., \overline{w}_n\}$$

$$\overline{w}_j \geq 1.0$$

Importance indices are defined as stochastic variable.

$g_j(a_i)$ is the evaluation of criterion g_j for alternative a_i.

Let ***a*** and ***b*** be two alternatives such that $a, b \in A$. Hence, we can define the following relations:

- aPb Where alternative a is strongly preferred to b
- aQb where alternative a is weakly preferred to b
- aIb Where alternative a is indifferent to b
- aSb where alternative *a* outranks alternative *b* that means “*a* is at least as good as *b*”

Like ELECTRE III, ELECTRE-SS defines two thresholds for each criterion:

- Indifference threshold $\overline{q}_j$
- Preference threshold $\overline{p}_j$

A third threshold *v* (veto threshold) may also be defined. However, this will not be considered in this implementation.

These thresholds are defined as stochastic variables and can vary along the scale of the criteria value. Hence we can write the preference and indifference relations as follows:

$$aSb^j \Leftrightarrow g_j(a) > g_j(b) + \overline{q_j}$$
$$aIb^j \Leftrightarrow \left| g_j(a) - g_j(b) \right| \leq \overline{q}_j$$

However, the above definition of the outranking relations fails to simulate the transition stage between a clear preference and indifference. Hence, the idea of weak and strong preference comes into effect:

$$a\overline{P}b^j \Leftrightarrow g_j(a) > g_j(b) + \overline{p}_j$$
$$a\overline{Q}b^j \Leftrightarrow \overline{q}_j < g_j(a) - g_j(b) \leq \overline{p}_j$$
$$a\overline{I}b^j \Leftrightarrow \left| g_j(a) - g_j(b) \right| \leq \overline{q}_j$$

Both values of *w* and threshold values (*p* and *q*) are defined to accommodate stakeholder evaluations and level of confidence in their evaluations. When the number of stakeholders is significantly large a probability distribution function (pdf) can be built to represent the entire spectrum of evaluations. However, when the number of stakeholders is not large enough to derive a pdf then the lowest value and the highest value are taken and a normal distribution is considered for evaluating the results in between the min-max range. Hence we can define the following:

$$\overline{w_j} = w_j^{\min} + (w_j^{\max} - w_j^{\min}) \times \Delta_j^w$$

Δ is the probability distribution function fit for the importance index of criteria *j*.

Similarly we define

$$\overline{q}_j = q_j^{\min} + (q_j^{\max} - q_j^{\min}) \times \Delta_j^q$$

And

$$\overline{p}_j = p_j^{\min} + (p_j^{\max} - p_j^{\min}) \times \Delta_j^p$$

Where Δ_j^q and Δ_j^p are the probability distribution functions of the indifference and preference thresholds for criteria *j* respectively.

Constructing the outranking relation

To construct the outranking relation we follow the procedures given in ELECTRE III which requires the construction of a *credibility index* $\rho(a,b)$ for the outranking relation *aSb*. The credibility index is defined using both a comprehensive *concordance index C(a,b)* and a discordance index $d_j(a,b)$ for each criterion $g_j \in G$.

Following the definition in ELECTRE III we calculate the partial concordance index

$$\bar{c}_j(a,b) = \begin{cases} 1, & g_{j(a)} + \bar{q}_j \geq g_j(b) \\ 0, & g_j(a) + \bar{p}_j \leq g_j(b) \\ \dfrac{\bar{p}_j + g_j(a) - g_j(b)}{\bar{p}_j - \bar{q}_j}, & otherwise \end{cases} \text{, where } j=1,..,n$$

The comprehensive concordance index is then calculated as follows:

$$\bar{C}(a,b) = \frac{1}{\bar{K}} \sum_{j=1}^{n} \bar{w}_j \times \bar{c}_j(a,b), \text{ where } \bar{K} = \sum_{j=1}^{n} \bar{w}_j$$

Since we are not considering a veto threshold, the discordance index is zero for all criteria. Therefore, the credibility index $\bar{\rho}(a,b)$ in this case is equal to the comprehensive concordance index $\bar{C}(a,b)$.

Exploitation of the outranking procedure

To exploit the outranking matrix, two complete pre-orders are constructed

- $\bar{Z}_1$, a descending distillation
- $\bar{Z}_2$, an ascending distillation

Where

$$\bar{Z}_1 = \{\bar{z}_{1,1}, \ldots, \bar{z}_{1,l}, \ldots, \bar{z}_{1,k}\}$$

$$\bar{Z}_2 = \{\bar{z}_{2,1}, \ldots, \bar{z}_{2,l}, \ldots, \bar{z}_{2,k}\}$$

$\bar{z}_{1,l}$, $\bar{z}_{2,l}$ are the number of times alternative a_i ranked in the k^{th} order in the descending and ascending distillations respectively.

Second we build two complete pre-orders Z_1, Z_2 such that

$$Z_1 = \bar{z}_{1,1} + \sum_{l=1}^{k} -l \times \bar{z}_{1,l}$$

$$Z_2 = \overline{z}_{2,1} + \sum_{l=1}^{k} - l \times \overline{z}_{2,l}$$

Finally a partial order is constructed as follows

$$Z = Z_1 \wedge Z_2$$

This algorithm allows decision makers to view the performance of different alternatives under uncertainty conditions.

Application

In this section we present an application of ELECTRE-SS method to a case study originally published in chapter 6 in Rogers et al. (2000).

Case study background

The Federal Agency for the Environment in Switzerland invited a group of cantons from the eastern region of Switzerland to constitute a working group to decide on the ‘optimum waste strategy for the region’. The region was divided into 4 zones for planning purposes. Eleven strategic options (table 1) were identified for further assessment against eleven environmental, economic, political and technical criteria (table 2).

Four major criteria categories are considered in the decision making: Environmental criteria (C1), Economic (C2), Technical (C3) and Political (C4). Each criterion is further divided into sub-criteria as in table 2. The performance of each alternative for each criterion is summarised in table 3. Each canton was asked to assign its own weighting for each criterion. Table 4 shows the weightings of each criterion as indicated by each canton (stakeholder). To apply ELECTRE III, average weightings were derived for each criterion. Table 6 shows the results of the original ranking exercise.

Table 1: Options (Adapted from Rogers et al., 2000**)**

Option	**Description**
A1.1	Construction of 2 new WTE plants in Ticino. WTE plants at zones 1, 2 and 3 at maximum capacity No waste imported
A1.2	Same as A1.1 Provision for treatment of waste imported from Austria and Germany
A2.1	Reduction of the capacity WTE plants in zones 1 and 2 No new WTE plants in Ticino and Graubünden Waste from Ticino transported to zone 1 and 2 Waste from Graubünden transported to zone 3
A2.2	Same as A2.1 except Waste from Ticino transported to zone 1
A2.3	Same as A2.2 Provision for waste from Germany to be imported to zones 1 and 2
A2.4	Same as A2.3 . Provision for waste from Austria to be imported to zones 1 and 3.

A3.1	Reduction of capacities in Zones 1, 2 and 3. A new WTE plant in Ticino. A new WTE plant at Graubünden. 5 Ktons/year of waste to be transported from zone 4 to Ticino.
A3.2	Reduction in the capacity in zone 1. A new plant in Ticino. A new plant in Graubünden. 5 Ktons/yer of waste transported from zone 4 to Ticino. Importation of waste from Germany and Austria to zones 1, 2 and 3 for disposal.
A4.1	Zones 3 and 4 to be merged Waste transported from Graubünden to zone 3 for disposal A new plant to be Ticino 5 Ktons/year of waste to be transported from zone 4 to Ticino Capacity of WTE plants in zones 1, 2 and 3 reduced.
A4.2	Same as A4.1 Waste to be imported from Germany to zones 1 and 2 for disposal
A4.3	Same as A4.2 Waste to be imported from Austria to zone 1 for disposal.

Table 2: Criteria (Adapted from Rogers et al., 2000**)**

Criteria	**Description**	**Units**
C1.1	Distance of waste transportation	Kton.km/Year
C1.2	Energy use	GWh/year
C1.3	Impact of gas emissions	Pop.MtNOx/Yr
C2.1	Average treatment cost per region	SF/Ton
C2.2	Uniformity of treatment costs	%
C3.1	Adaptability to increase in waste production	Ktons/year
C3.2	Adaptability to decrease in waste production	Ktons/year
C3.3	Overcapacity	%
C4.1	Opposition of local pressure	Score (0 – 1.5)
C4.2	Opposition to the importation of foreign waste	Score (0 – 1.5)
C4.3	Dependency on supply of imported waste	Score (1 – 11)

Table 3: Performance matrix (Adapted from Rogers et al., 2000**)**

	C1.1	**C1.2**	**C1.3**	**C2.1**	**C2.2**	**C3.1**	**C3.2**	**C3.3**	**C4.1**	**C4.2**	**C4.3**
Options	Desc	Asc	Desc	Desc	Desc	Asc	Asc	Desc	Desc	Desc	Desc
A1.1	125	866	9.81	218	1.41	542	483	23	1.5	1	1
A1.2	11980	900	11.45	189	1.45	452	303	12	1.5	6	6
A2.1	31054	883	9.86	172	1.82	341	311	0	0	3	3
A2.2	28219	840	10.38	171	1.95	339	318	0	0	3	3
A2.3	31579	903	10.74	165	1.7	312	281	0	0	5	5
A2.4	39364	922	13.87	167	1.65	287	269	0	0	8	7
A3.1	125	769	9.33	182	1.64	458	180	0	1.5	1	1
A3.2	8075	896	9.82	172	1.7	408	121	0	1.5	6	6

A4.1	3089	770	9.39	177	1.9	430	228	0	1	2	2
A4.2	6449	766	7.22	172	1.65	401	157	0	1	4	4
A4.3	12074	897	10.61	169	1.65	378	162	0	1	7	6

Table 4: Criterion Weightings (Adapted from Rogers et al., 2000**)**

Criterion	**Zurich**	**Glarus**	**StGallen**	**Graubund**	**Thurgau**	**FAE**
C1.1	4.7	12.8	12.1	17.3	10.7	16
C1.2	12.2	3.5	5	7.9	6	3.3
C1.3	20	10.6	4.5	7.1	1.9	3.3
C2.1	11.6	17.7	13.8	19.3	19.3	9.7
C2.2	8.7	9	6.5	12	12.4	9.7
C3.1	13	1.2	9.8	5.3	16.2	16
C3.2	14.4	2.9	11.2	5.3	16.2	9.7
C3.3	6.8	10.2	9.3	12	6.8	16
C4.1	10.2	15.1	11.7	1.3	6.7	3.3
C4.2	8.2	9.3	8.4	1.3	1.9	3.3
C4.3	8.2	7.7	7.9	11.2	1.9	9.7
Total	100	100	100	100	100	100

Table 5: Indifference and Preference Thresholds (Adapted from Rogers et al., 2000**)**

Criterion	**Indifference Threshold (q)**	**Preference Threshold (p)**
C1.1	1000	2000
C1.2	10%	20%
C1.3	10%	20%
C2.1	5	10
C2.2	10%	20%
C3.1	10%	20%
C3.2	10%	20%
C3.3	2	4
C4.1	0.2	0.4
C4.2	0	1
C4.3	0	1

Table 6: ELECTRE III Ranking of Alternatives (Adapted from Rogers et al., 2000**)**

Rank	**Alternative**
1	A4.1, A3.1
2	A2.2
3	A4.2
4	A2.3
5	A1.1, A2.1, A4.3
6	A1.2, A3.2
7	A2.4

In our application of ELECTRE-SS we suggest that the values of *p* and *q* are not known for certain but fall in a range. The values of weightings given by different cantons differed significantly as obvious from Table 4. In our application of ELECTRE-SS we assume that the actual value for each weighting falls between the lowest and largest assigned values for each criteria. We run ELECTRE-SS with 10 000 simulations with each time changing the values for *p* and *q* randomly within the ranges as in Table 7 and the values for weightings as in Table 8.

Table 7: Indifference and preference threshold ranges

Criterion	**Indifference Threshold (q)**	**Preference Threshold (p)**
C1.1	0-1000	1000-2000
C1.2	5%-10%	10%-20%
C1.3	5%-10%	10%-20%
C2.1	0-5	5-10
C2.2	5%-10%	10%-20%
C3.1	5%-10%	10%-20%
C3.2	5%-10%	10%-20%
C3.3	0-2	2-4
C4.1	0 - 0.2	0.2 - 0.4
C4.2	0 - 1	1 - 2
C4.3	0 - 1	1 - 2

Table 8: Criteria Ranges

Criteria	**Lower Limit**	**Upper Limit**
C1.1	4.7	17.3
C1.2	3.3	12.2
C1.3	1.9	20
C2.1	9.7	19.3
C2.2	6.5	12.4
C3.1	1.2	16.2
C3.2	2.9	16.2
C3.3	6.8	16
C4.1	1.3	15.1
C4.2	1.3	9.3
C4.3	1.9	9.7

Results

The following four scenarios were run:

1- threshold values only were allowed to vary within the ranges in table 7 while the average weightings were used as in the original case
2- weighting were allowed to change within the ranges presented in table 8 while fixing the values of thresholds as in the original case
3- both thresholds and weightings were allowed to vary within the ranges as in table 7 and table 8

<u>Scenario 1:</u>

In this scenario we ran the method for 10,000 simulations allowing both the values of the indifference threshold *q* and the preference threshold *q* to change randomly between the lower

limit of the range to the maximum limit of the range. The results are presented in fig.1 and Table 9. The results show that when considering uncertainty in threshold values, alternative A4.2 occupies the first rank and A2.3 follows very closely. Alternative A3.1 and A4.1 which ranked first in the original study now share the fourth position along with alternative A1.1. Nevertheless, alternative A2.4 continues to rank the worst, in fact, A2.4 has never ranked better than third in any run. A close analysis of fig. 1 reveals that although alternative A4.1 has a better overall ranking score and better performance for the first rank, alternative A2.3 had never ranked worse than third in any run while A4.2 had ranked third or better only 93% of the time.
Clearly, the results show that ranking of alternatives is sensitive to the threshold values.

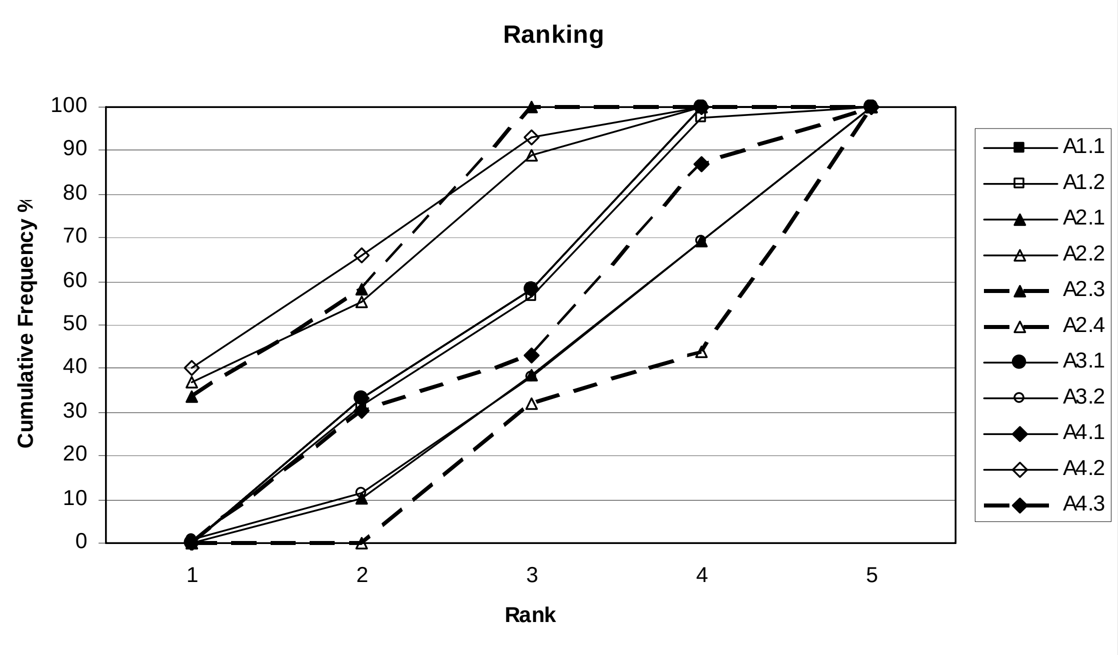

Fig. 1: Cumulative Ranking under uncertain threshold values

Table 9: Final Ranking under uncertain threshold values

Alt Name	Cumulative Score	Rank
A4.2	-40152	1
A2.3	-41618	2
A2.2	-43786	3
A1.1	-61712	4
A3.1	-61712	4
A4.1	-61712	4
A1.2	-62832	5
A4.3	-67934	6
A3.2	-76068	7
A2.1	-76344	7
A2.4	-84852	8

Scenario 2:

In this scenario we assumed that threshold values were known with precision while criteria weightings were uncertain. The results are shown in fig. 2 and Table 10.

Alternative A2.3 and A4.2 continue to lead the alternatives' ranking while A2.4 remains the least favoured alternative. However, A2.3 out-performs A4.2. Alternative A2.3 ranks third or better 50% of the time while A4.2 ranks third or better only 47% of the time. Alternatives A3.1 and A4.1 performed moderately with A3.1 and A4.1 ranking third or better in 46.8% and 46.85% of the time respectively.

Again we see that alternative's ranking is sensitive to the value of the criteria weighting.

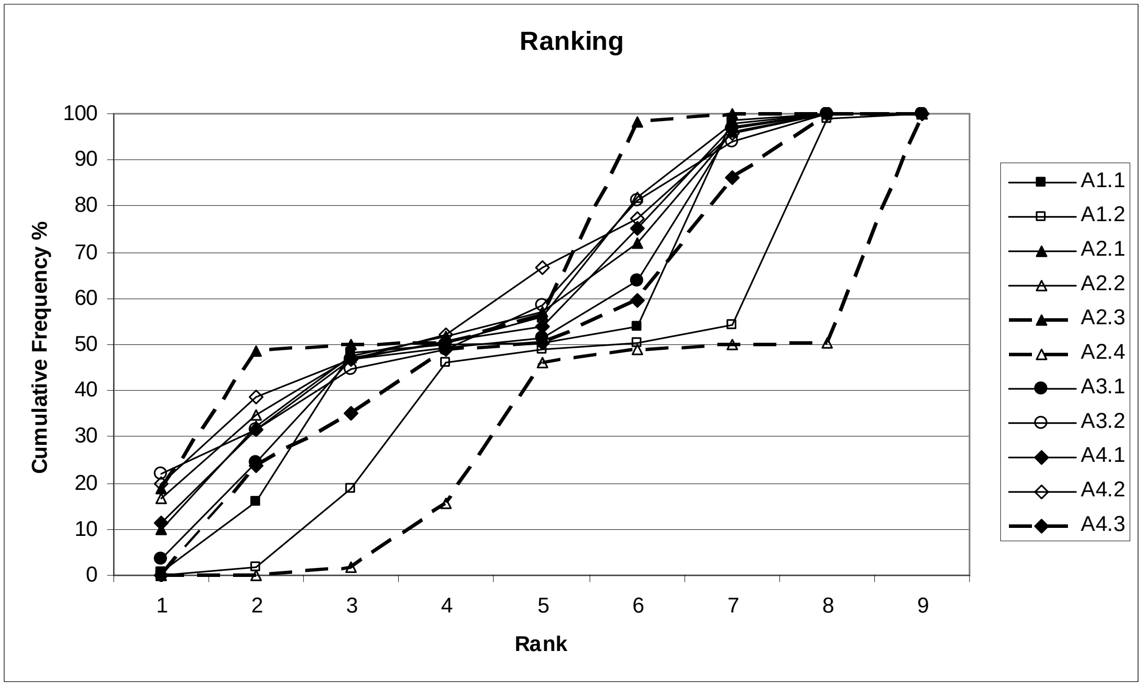

Fig. 2: Cumulative ranking under uncertain weightings

Table 10: Final Ranking under uncertain weighting

Alt Name	Cumulative Score	Rank
A2.3	-75511	1
A4.2	-80629	2
A2.2	-83027	3
A3.2	-83877	4
A2.1	-86609	5
A4.1	-86641	6
A3.1	-92785	7
A1.1	-96425	8
A4.2	-99239	9
A1.2	-116143	10

A2.4	-137495	11

Scenario 3:

When considering uncertainty in criteria weights and threshold values simultaneously alternatives A4.2 and A2.3 remain in the lead. However, alternative A4.2 performs slightly better than A2.3 with A4.2 ranking third or better 49.9% of the time and A2.3 ranking third or better 49.8% of the time. Alternatives A3.1 and A4.1 continue to perform moderately with A3.1 is significantly better than A4.1 (49.94% vs 48.3%.third rank or better respectively). Alternative A2.4 remains the least performing alternative.

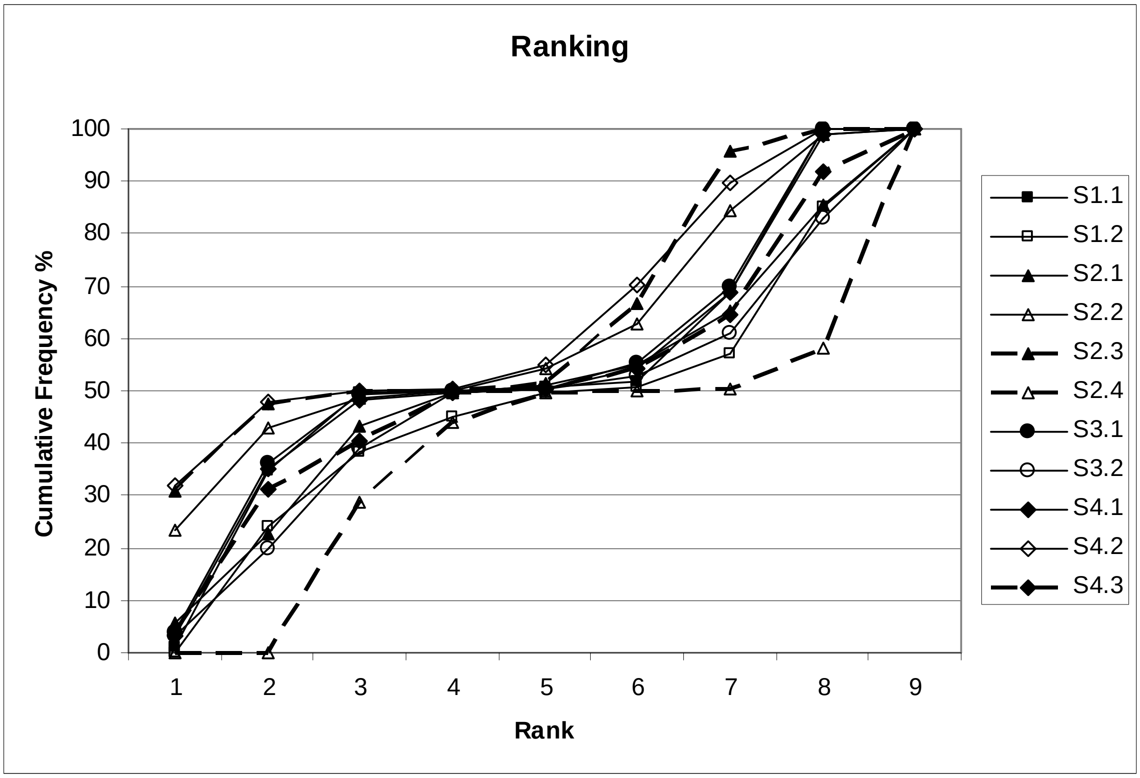

Fig 3: Cumulative ranking distribution under uncertain weightings and thresholds

Table 11: Final ranking under uncertain weightings and thresholds

Alt Name	Cumulative Score	Rank
A4.2	-80938	1
A2.3	-81574	2
A2.2	-87164	3
A3.1	-96882	4
A4.1	-98230	5
A1.1	-98566	6
A4.3	-102782	7
A2.1	-104286	8
A3.2	-108310	9

A1.2	-110090	10
A2.4	-123828	11

Conclusion

The above results demonstrate that final ranking of alternatives is sensitive to both threshold and criteria weight values. The results also indicate that the final ranking is more sensitive to criteria weights than threshold values. We also find that average weights are not necessarily good estimates of criterion weights. In fact, scenario 2 shows significant discrepancies between the results obtained using average weight and actual weight ranges.

Using ELECTRE-SS method has the advantage of assessing the performance reliability of the selected alternative which is not possible when using the deterministic ELECTRE III method. It also allows for close inspection of each alternative's performance, hence decisions may include alternatives that otherwise may be excluded if deterministic parameters where used. Finally, the method provides easy presentation of results in tabular format that gives the decision maker a clear ranking which can be further inspected using the graphical presentation mode.

Evaluating municipal solid waste management alternatives usually involves a great deal of uncertainty especially when considering social and environmental criteria. We intend to incorporate this method in a framework that allows the evaluation of alternatives utilising LCA methodology and ordinal social criteria under uncertainty.

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